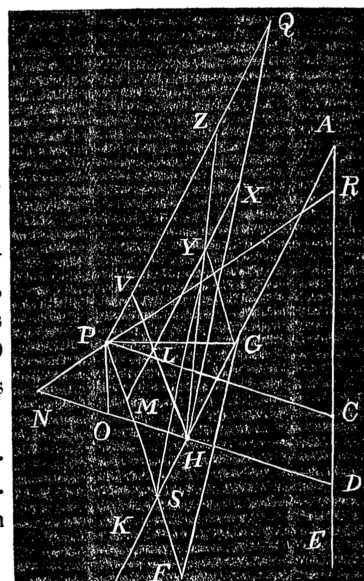
SOLUTIONS OF PROBLEMS IN NUMBER THREE.

Solutions of problems in No. 3 have been received as follows:

From Prof. W. P. Casey, 437, 438; Prof. H. T. Eddy, 440; Prof. A. Hall, 440; E. H. Moore, Jr., 438; Ernest G. Merritt, 438.

437. *By Prof. Casey.*—"AE, AK are two indefinite given straight lines, C and H given points in them, and P a given point in their plane. Req'd to draw through P two straight lines, PB, PD, intersecting AE and AK in R and S, respectively, and containing a given angle RPS, so that $RC \times SH$ may be equal to a given magnitude."

SOLUTION BY THE PROPOSER.



Let AE, AK be two indefinite straight lines given in position; C, H, given points in them, and P a giv'n point in their plane; and let PR, PS be drawn meeting them in R, S, so that $RC \times HS$ may be $=$ a given space, A^2; and the $\angle RPS =$ a given angle.

Analysis.—Join PC; it is given in position. Through P, H, draw PO, DHO, parallel to AE, PC, respectively; $\therefore$ O is a given point. Produce RP to meet DO in N. Now $RC \times ON = PC \times CO$ and is therefore given, and $RC \times HS$ is given; therefore the ratio of ON to HS is given. Angle RPS is given, $\therefore$ $\angle NPS$ is given.

Make $\angle OPG = \angle NPS$, then PG is in position and G is a given point. Make $\angle PGF = PON =$ a given angle, therefore GF is in position, and triangles PON and PGF are similar, and $ON : GF :: PO : PG$, i. e., in a given ratio, and the ratio of $ON : HS$ is given, $\therefore$ the ratio of $GF : HS$ is given.

Through H draw HQ parallel to GF, $\therefore$ HQ is in position. Draw SY parallel to HQ, and GY parallel to PS, meeting SY in Y. Through Y draw XYM parallel to AK, and through P draw PQ parallel to AK, $\therefore$ PQ is in position, and Q is a given point. Join HY and produce it to meet PQ in Z and draw HL parallel to PS. Then $GF = SY = HX$, and

$HS = XY$; and as the ratio of $GF : HS$ is given, $\therefore$ the ratio of HX to XY is given; hence $HQ : QZ$ is given and HQ is given, $\therefore$ QZ is given and Q is a given point, $\therefore$ Z is a given point and and PZ is a given line. But $QZ \times YL = XY \times ZV$, and $YL = GH$, a given line, also $XY = HS = VP$, $\therefore$ $ZV \times VP = QZ \times GH$, a given space, and ZP is given, $\therefore$ V is a given point and VH is in position, hence PS is in position and so is PR, and the problem is solved geometrically.

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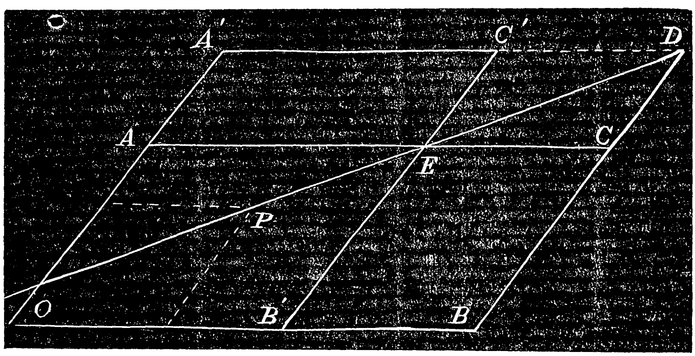
438. *By Prof. F. H. Loud.*—"Two equiangular parallelograms, $OACB$ and $OA'C'B'$ are so placed that the equal angles AOB and $A'OB'$ coincide. The sides of the former figure are constant, those of the latter are variable, subject to the condition $A'O + OB' : AO + OB ::$ *area* $OA'C'B'$: *area* $OACB$. $A'C'$ meets BC in D, and $B'C'$ meets AC in E. Show that DE passes through a fixed point, and determine the point."

SOLUTION BY ERNEST G. MERRITT, CORNELL UNIV., ITHACA, N. Y.



Put $OB = a$ $OA = b$, coordinates of the point C, and let the coordinates of C' be $OB' = x'$ and $OA' = y'$.

Then, from the conditions of the problem,

$$x' + y' : a + b = x'y' \sin AOB : ab \sin AOB;$$

$$\therefore (a+b)x'y' = ab(x'+y'), \tag{1}$$

whence
$$y' = \frac{abx'}{(a+b)x' - ab}. \tag{2}$$

The coordinates of D are (a, y'), and of E, (x', b). Substituting in the equation of a line through two points (x_1, y_1) and (x_2, y_2)

$$\left[y - y_1 = (x - x_1)\frac{y_2 - y_1}{x_2 - x_1} \right],$$

we obtain $y - b = \left(\dfrac{x - x'}{a - x'}\right)\left(\dfrac{abx'}{(a+b)x' - ab} - b\right)$, which may be written

$$y - b = (x - x')\frac{\dfrac{ab}{a+b} - b}{\dfrac{ab}{a+b} - x'}. \tag{3}$$

Equation (3) represents a line passing through E and the fixed point $[ab\div(a+b),\ ab\div(a+b)]$, but it is also the eq'n of the line DE. Henc the line DE passes through the point P whose coordinates are $ab\div(a+b)$ and $ab\div(a+b)$.

Cor. From eq'n (1) it is seen that the locus of C is an hyperbola whose center is at P and whose asymptotes are parallel to OB and OA.

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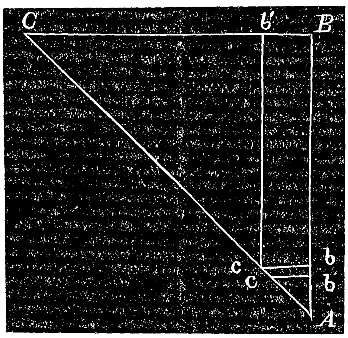
439. *By Prof. Nicholson.*—"Solve geometrically the following:

On a line whose length is a are two points x distance apart; what is the average value of x?"

No solution of this problem has been received. It may however readily be solved as follows:

Let x be any distance $Ab = bc$, estimated from A $(AB=a)$. Then is $(a-x)x =$ area $bBb'c =$ the sum of the distances of the two points, which being taken for all values of x, from 0 to a, will represent the volume of a triangular pyramid whose base is the area of the triangle BCD (D being vertically above B), and altitude, $BA = a$, and is, therefore, $\frac{1}{6}a^3$. This divided by $\frac{1}{2}a^2$, the whole num. of positions, gives $\frac{1}{3}a$ for the required average.



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440 *Selected.*—"A lamina is bounded on two sides by two similar ellipses, the ratio of the axes in each being m, and on the other two sides by two similar hyperbolas, the ratio of the axes in each being n. These four curves have their principal diameters along the coordinate axes. Prove that the product of inertia about the coordinate axes is

$$\frac{(\alpha^2-\alpha'^2)\,(\beta^2-\beta'^2)}{4(m^2+n^2)}$$

where $\alpha\alpha'$, $\beta\beta'$ are the semi-major axes of the curves." (Routh's Rigid Dynamics.)

SOLUTION BY PROF. HENRY T. EDDY, PH. D.

$F = \int\int xydxdy.$ $\therefore$ by Tod. Int. Cal., Art. 239,

$$F = \pm\int\int\frac{xydudv}{\frac{du}{dx}\frac{dv}{dy}-\frac{du}{dy}\frac{dv}{dx}}$$

But $u = x^2 + m^2y^2$, $v = x^2 - n^2y^2$; therefore

$$F = \pm \int_{a'^2}^{a^2}\int_{\beta'^2}^{\beta^2} \frac{xydudv}{4xy(m^2+n^2)} = \frac{(a^2-a'^2)(\beta^2-\beta'^2)}{4(m^2+n^2)}.$$

SOLUTION BY PROF. ASAPH HALL.

The ratio of the axes of the curves being given, we may write the eq'ns

$$x^2 + m^2y^2 = u; \qquad x^2 - n^2y^2 = v.$$

The product of inertia is given by the integral $\int xydxdy$, and we have to transform this to the variables u and v. The partial derivatives are,

$$\frac{dx}{du} = \frac{n^2}{2x(m^2+n^2)}; \qquad \frac{dx}{dv} = \frac{m^2}{2x(m^2+n^2)};$$

$$\frac{dy}{du} = \frac{1}{2y(m^2+n^2)}; \qquad \frac{dy}{dv} = \frac{-1}{2y(m^2+n^2)}.$$

Forming the known determinant for the transformation we have,

$$\int xy\,dxdy = \frac{1}{4(m^2+n^2)}\int du\,dv = \frac{(a^2-a'^2)(\beta^2+\beta'^2)}{4(m^2+n^2)};$$

since the limits of u are a^2 and a'^2, and of v, β^2 and β'^2.

REMARKS ON "NEW RULE FOR CUBE ROOT."—We published on page 98, No. 3, what purports to be a new Rule for Cube Root, and were not aware, at the time, that substantially the same rule had been published before; and we have no doubt the author believed it to be new, and original with him.

The same Rule, in effect, may be found at page 32, Vol. I of the *Mathematical Monthly*, published in Nov., 1859, at Cambridge, Mass. The editor (J. D. Runkle) there says, "In the Nouvelles Mathe'matiques for January, 1858, we find the following method for extracting the cube root of numbers, which ought, on account of its easy application, to be generally used. The editor remarks, in the April number, that the method had previously been given in a work entitled *Calcul pratiques*, in which it is claimed as new. The reader will find the same procees, entitled a new method, in the American edition of Young's Algebra, published as long ago as 1832. It may also be found in some of our arithmetics; and many teachers undoubtedly already know and use it."

PROBLEMS.

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441. *By Wm. Hoover, A. M., Dayton, Ohio.* —A cone revolves around its axis with a known angular velocity. The altitude begins to diminish